Objectives

1. To show how to add forces and resolve them into components using the parallelogram law.
2. To express force and position in Cartesian vector form and explain how to determine the vector’s magnitude and direction.

Definitions

Scalar - A quantity characterized by a positive or negative number is called a scalar. Examples of scalars used in Statics are mass, volume or length.

Definitions

Vector - A quantity that has both magnitude and a direction. Examples of vectors used in Statics are position, force, and moment.

Symbols

Vectors are denoted by a letter with an arrow over it or a boldface letter such as **A**

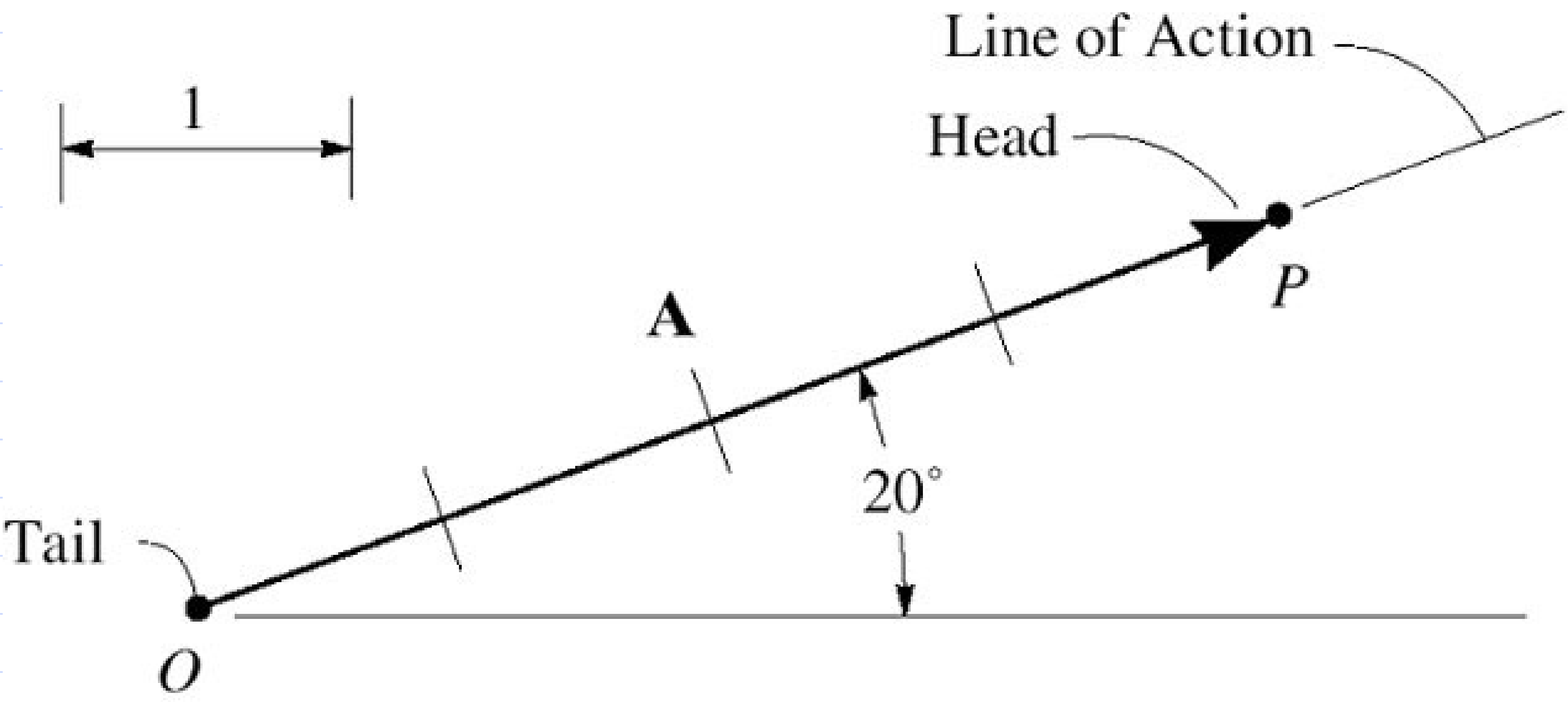
Symbols used for vectors:

$$\vec{A} \text{ or } \vec{A}$$

Denote magnitude by:

$$|\vec{A}| \text{ or } A$$

Vector Definitions



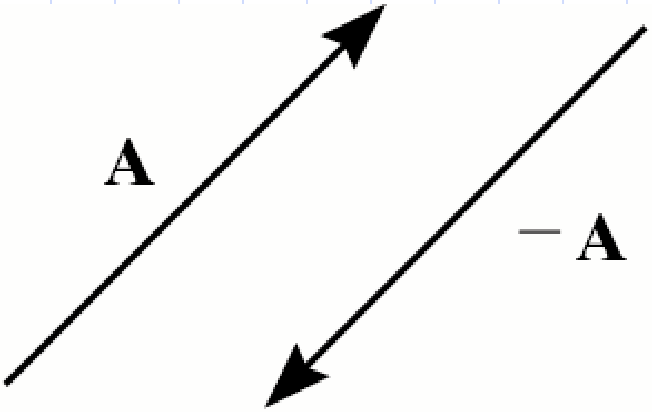
Magnitude and Multiplication of Vector by Scalar

- The magnitude of a quantity is always positive.
- If m is scalar quantity and it z multiplied to a vector **A** we get m**A.**
- **What does it mean?**

- m**A** is vector having same direction as A and magnitude equal to the ordinary scalar product between the magnitude of m and **A.**

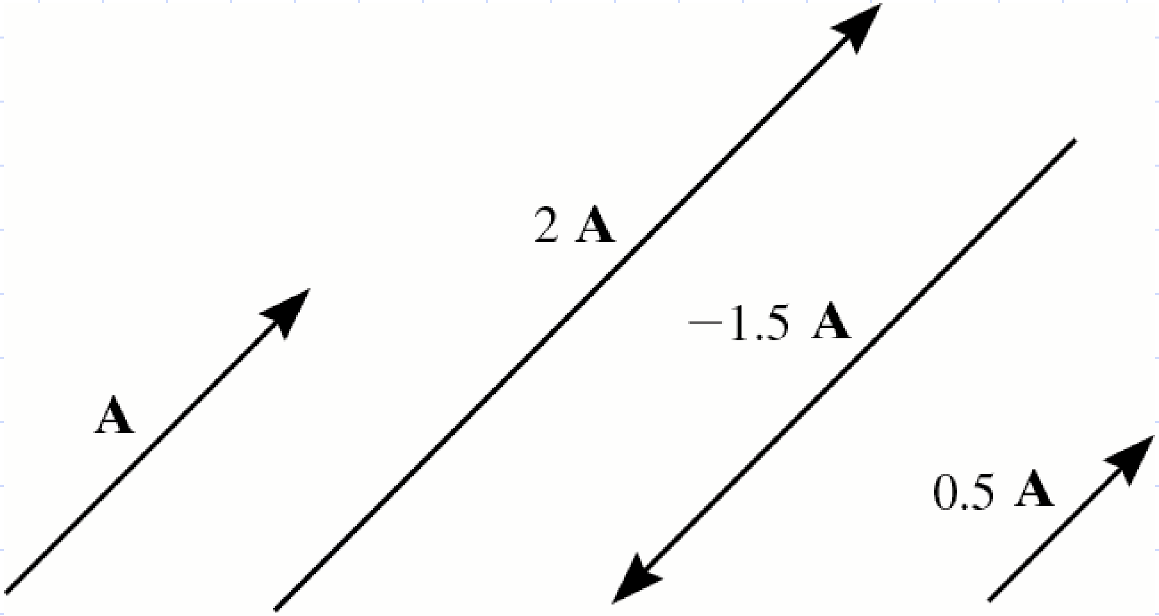
- **what happens if m is negative?**

Scalar Multiplication



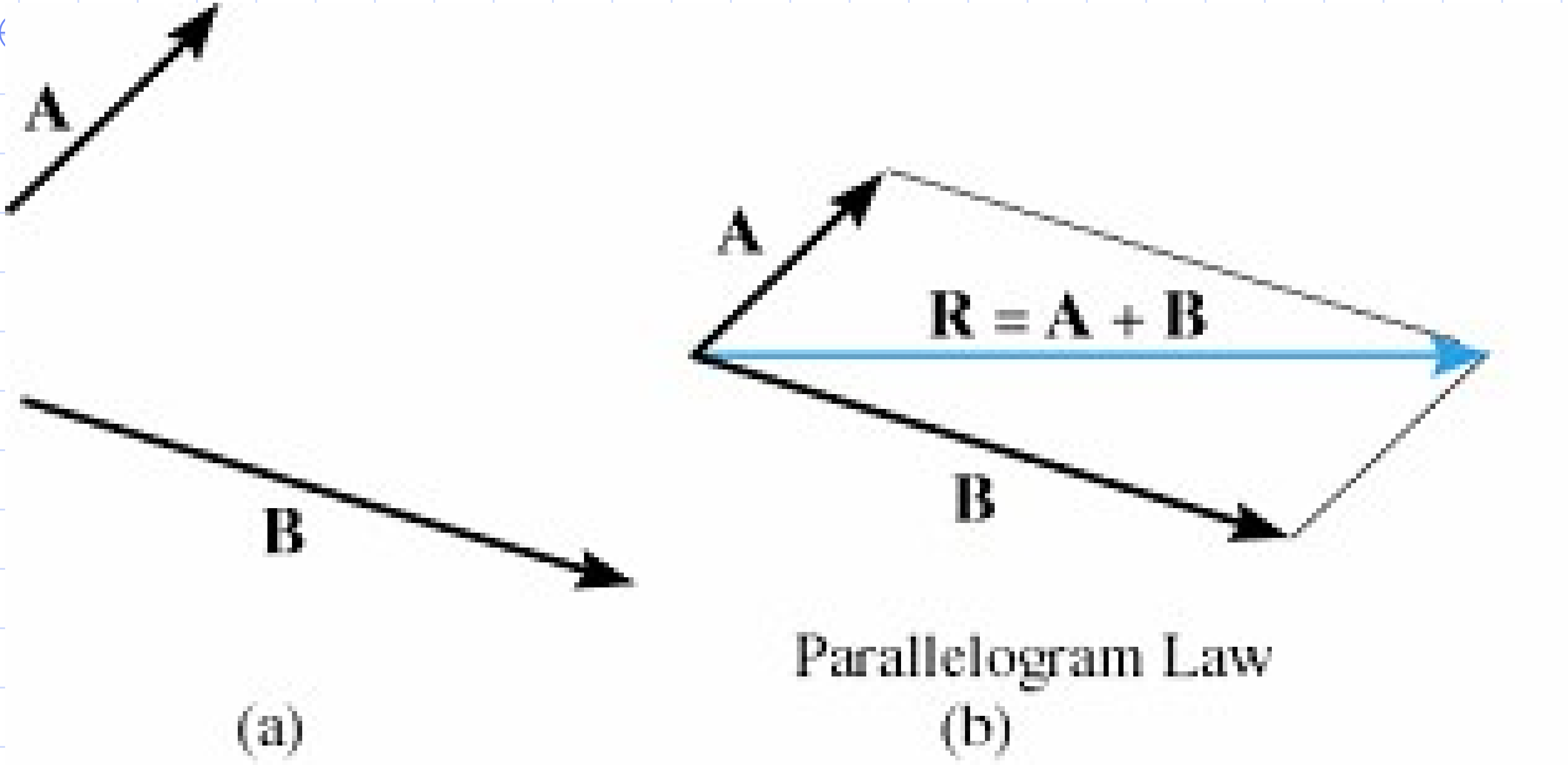
Vector **A** and its negative counterpart

Scalar Multiplication



Scalar Multiplication and Division

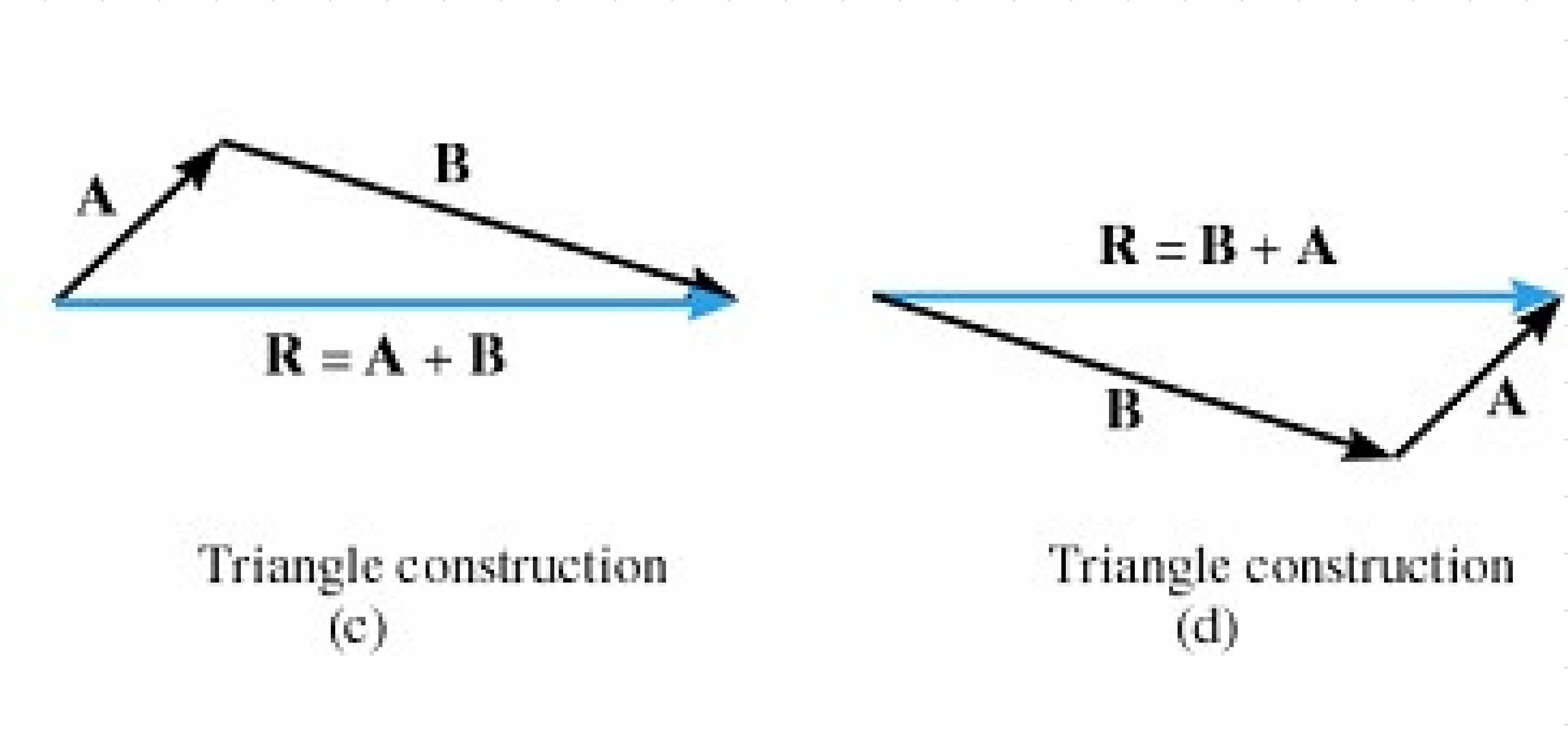
Vector Addition



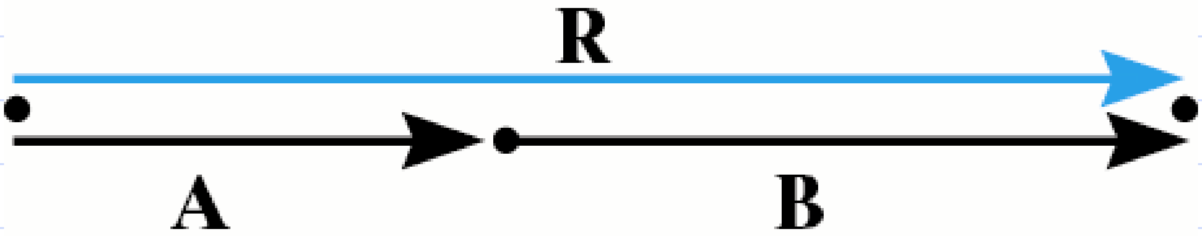
- Vector addition is commutative and associative.

- **How?**

Vector Addition



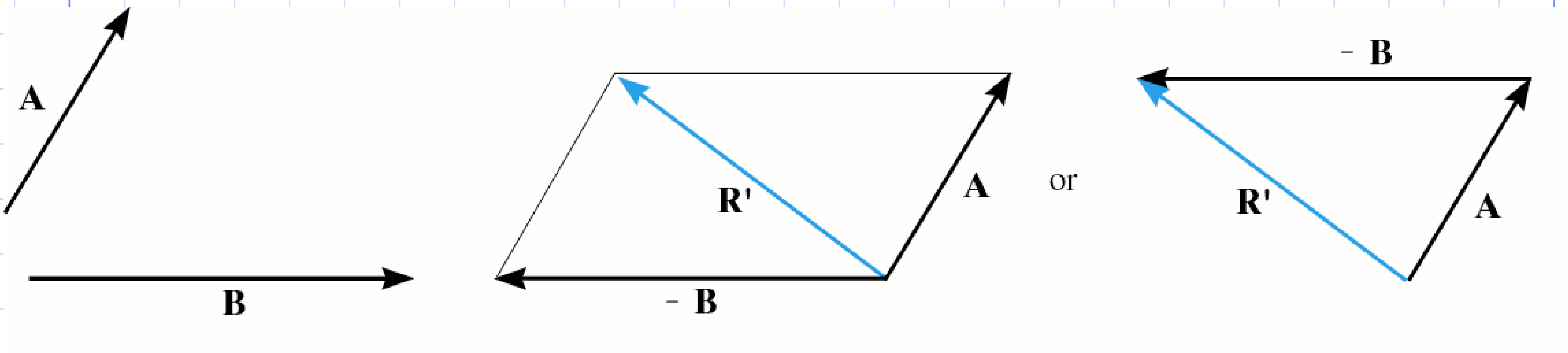
Vector Addition



$$R = A + B$$

Addition of collinear vectors

Vector Subtraction

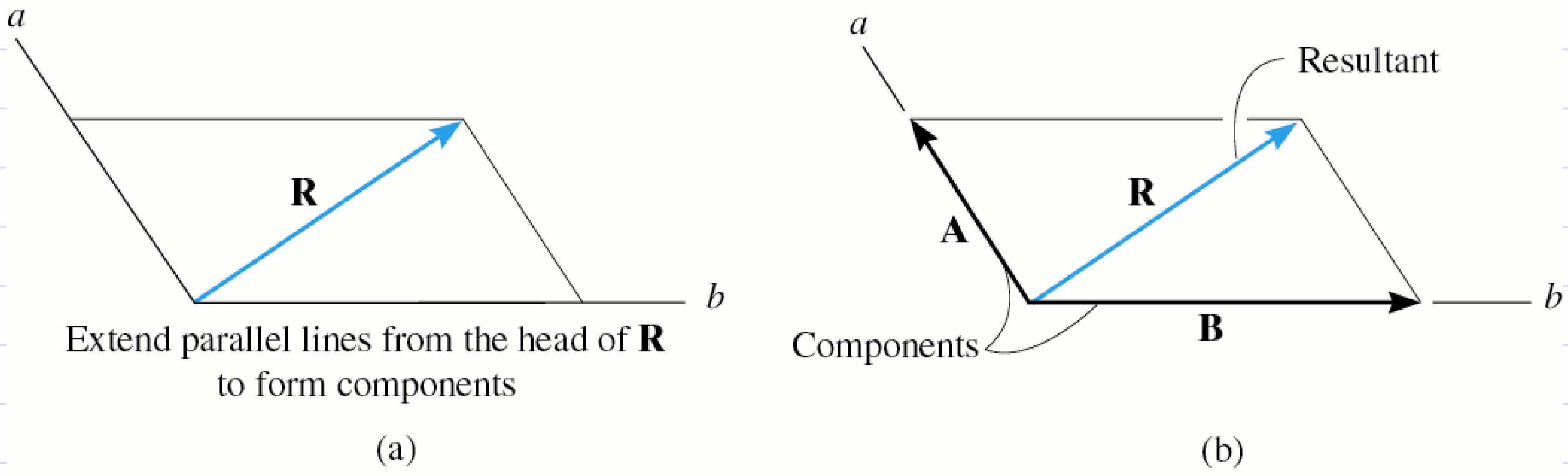


Parallelogram law

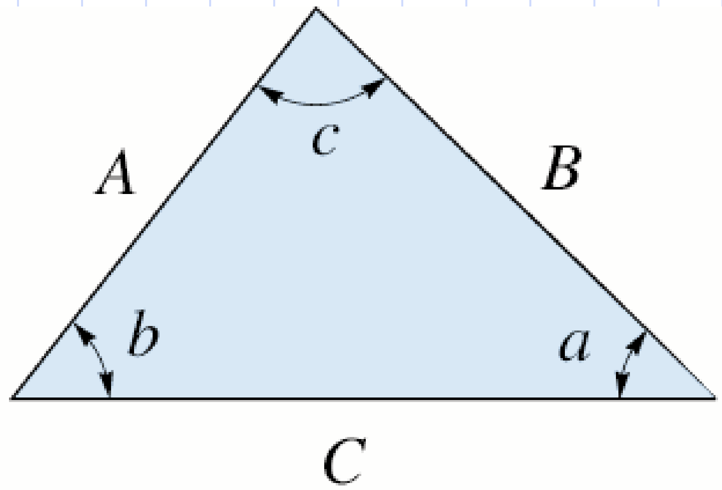
Triangle construction

Vector Subtraction

Resolution of a Vector



Resolution of a vector



Sine law:

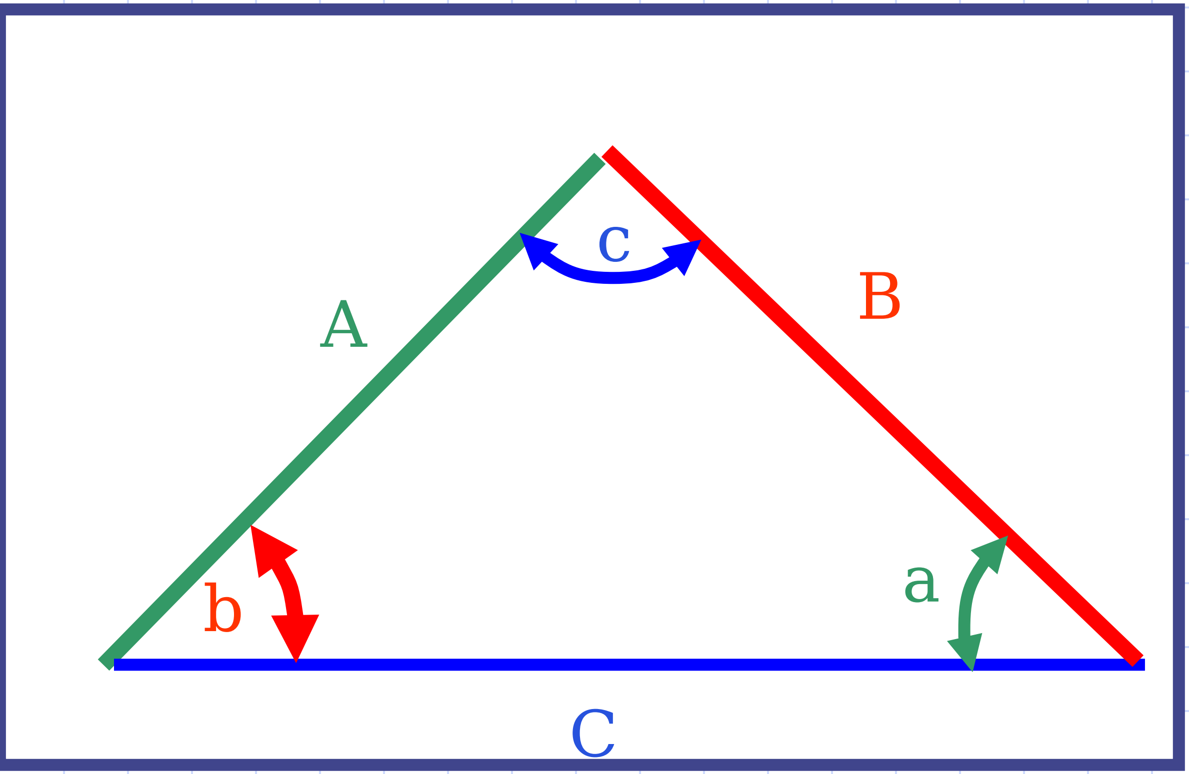
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2\,A\,B\cos c}$$

Figure 02.09

Trigonometry



Law of Sines:

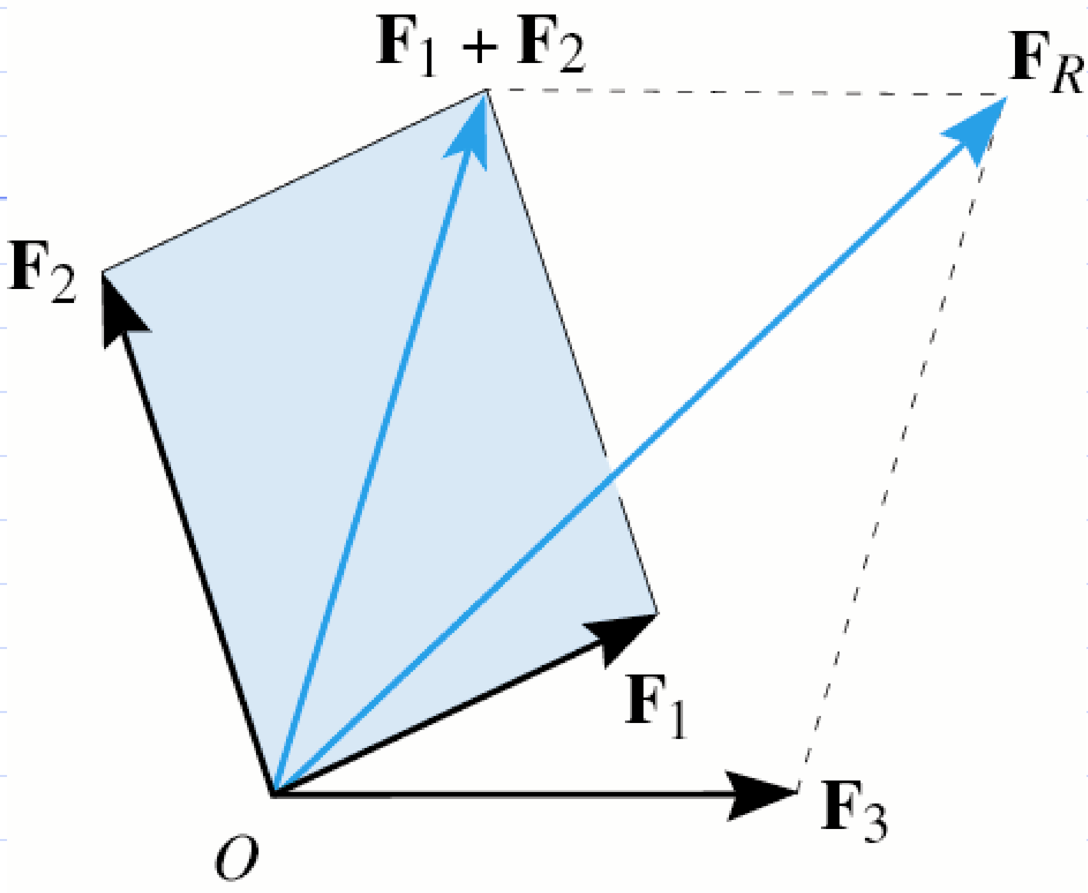
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Law of Cosines

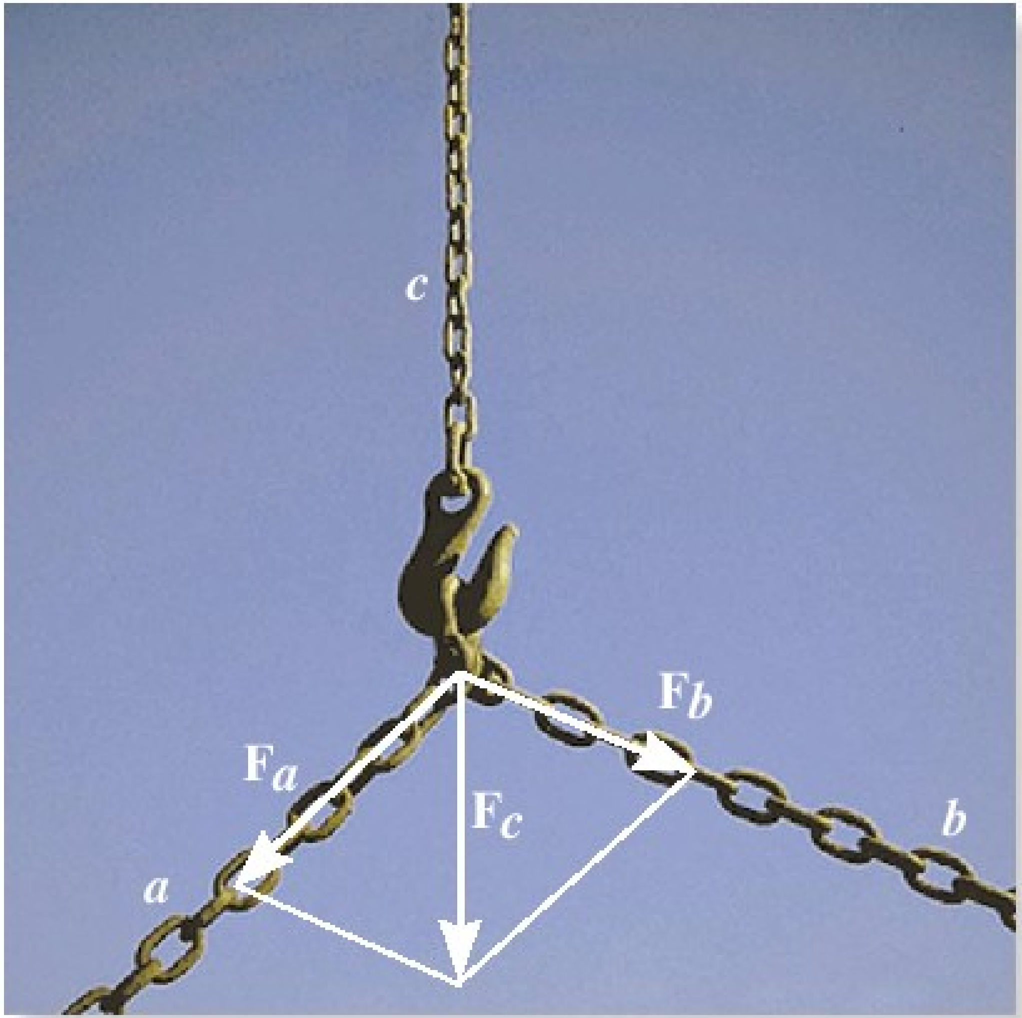
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Force

1. Force is a Vector Quantity
2. Forces Add as Vectors



$\mathbf{F}_1 + \mathbf{F}_2$
$\mathbf{F}_R$
$\mathbf{F}_2$
$\mathbf{F}_1$
$\mathbf{F}_3$
O


c
Fb
Fa
Fc
a
b

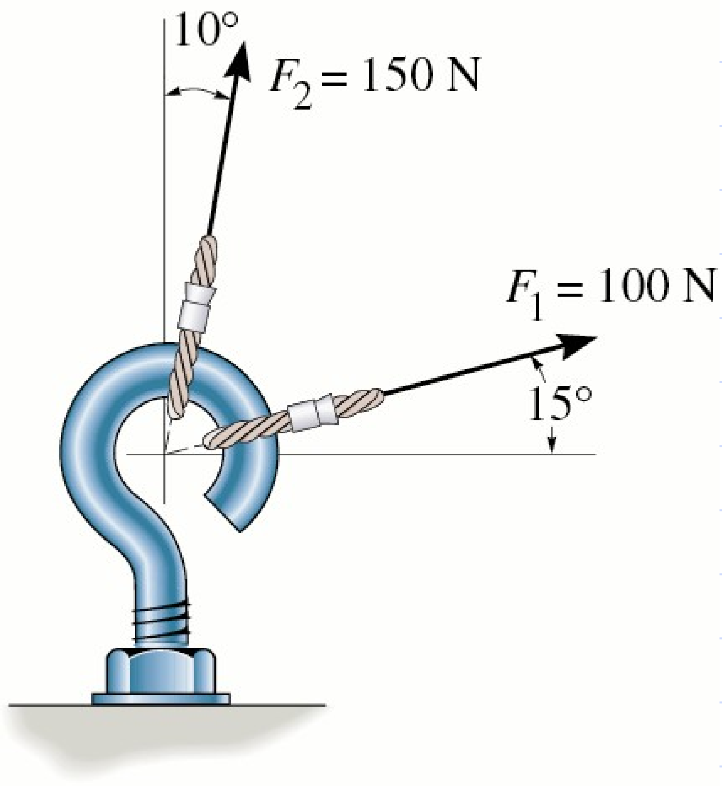
Parallelogram Law

1. Make a sketch showing vector addition using the parallelogram law.
2. Determine the interior angles of the parallelogram from the geometry of the problem.
3. Label all known and unknown angles and forces in the sketch.
4. Redraw one half of the parallelogram to show the triangular head-to-tail addition of the components and apply laws of sines and cosines.

Important Points

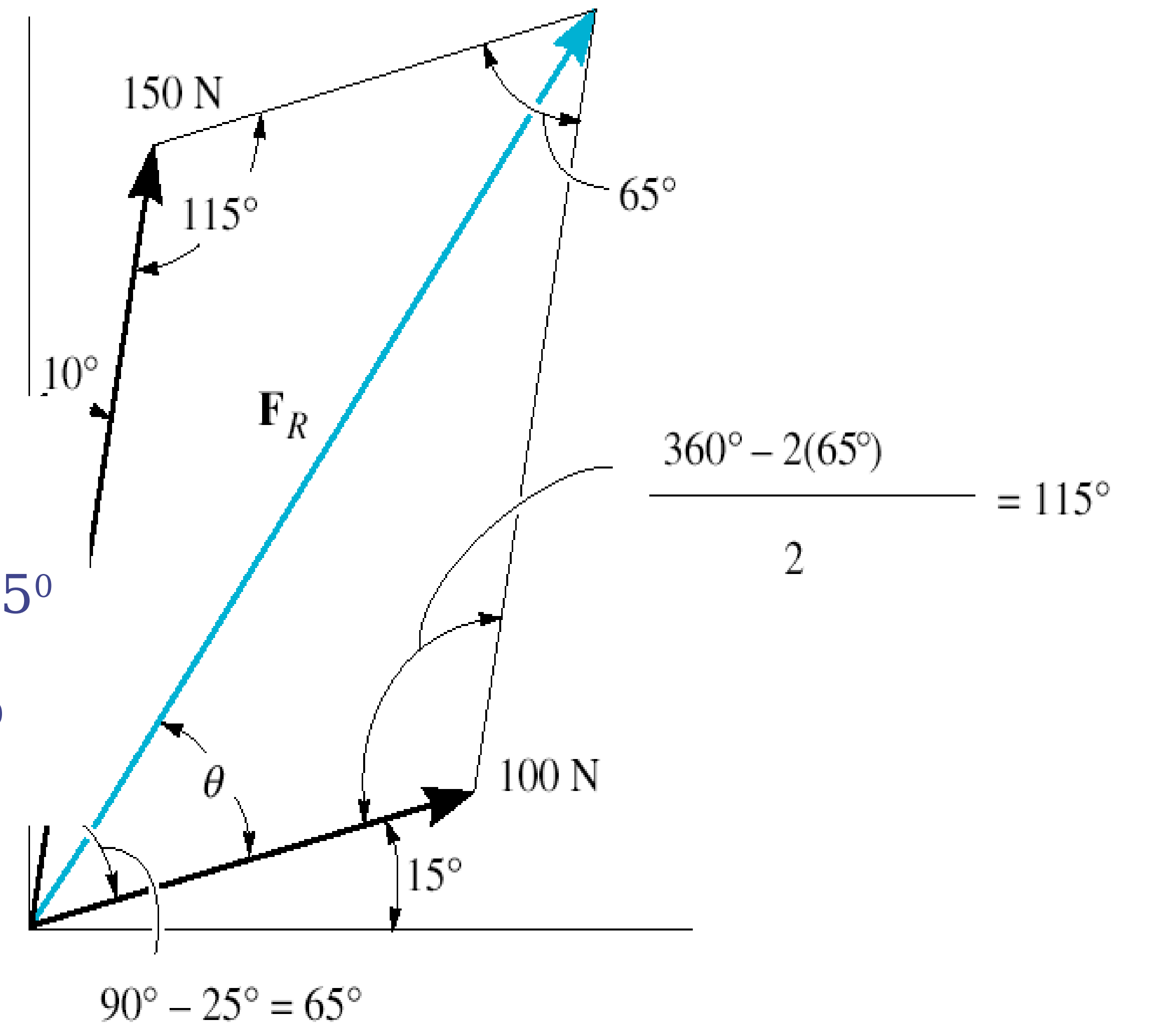
1. A scalar is a positive or negative number.
2. A vector is a quantity that has magnitude, direction, and sense.
3. Multiplication or division of a vector by a scalar will change the magnitude. The sense will change if the scalar is negative.
4. If the vectors are collinear, the resultant is formed by algebraic or scalar addition.

Example



The screw eye in the figure at the left is subjected to two forces $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the magnitude and direction of the resultant force.

Parallelogram Law

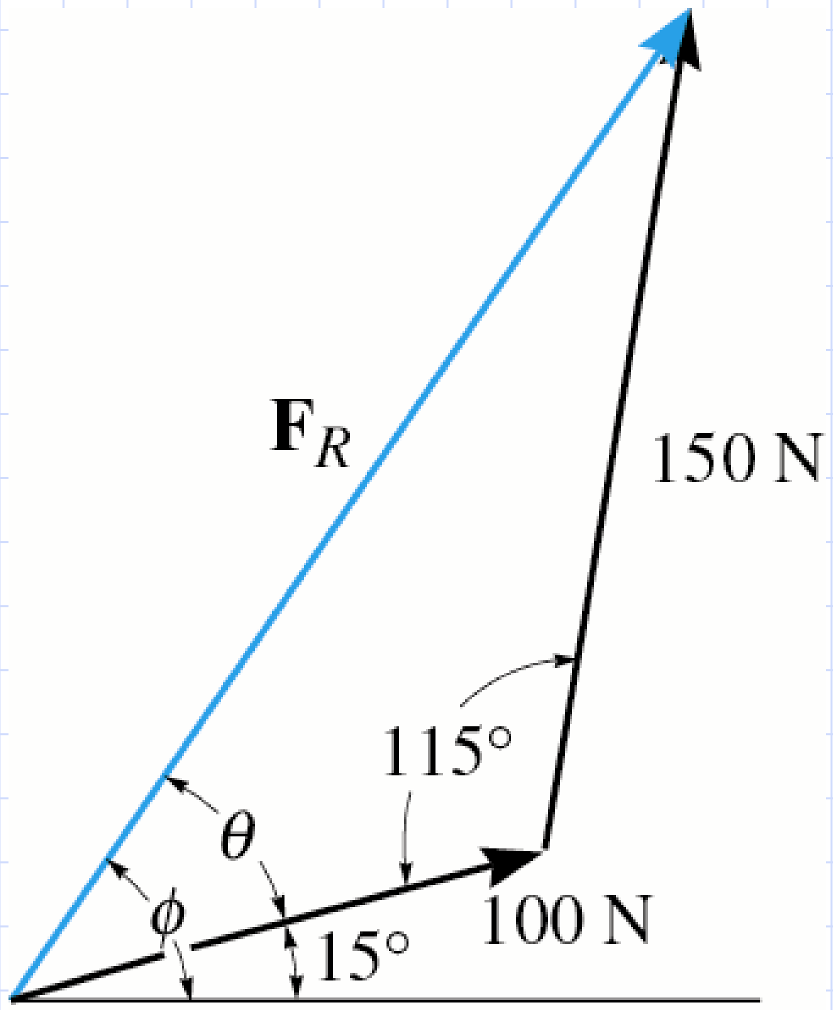


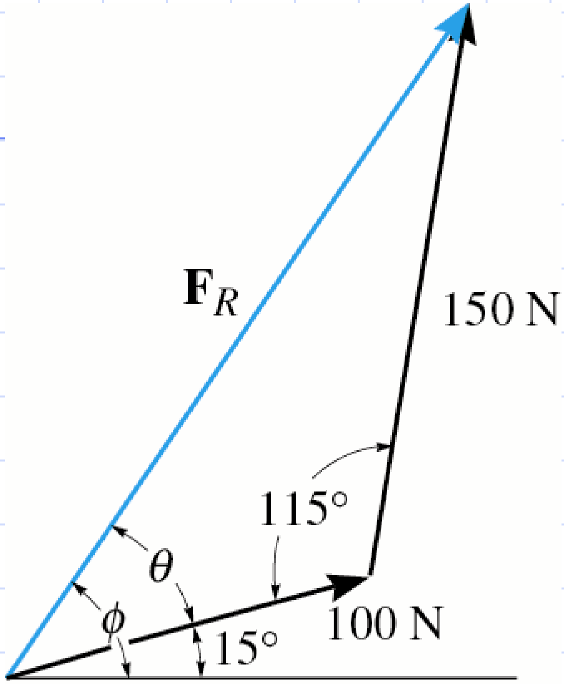
alculate angles

ngle COA = $90^0 - 15^0 - 10^0 = 65^0$

ngle OAB = $180^0 - 65^0 = 115^0$

Constructio n





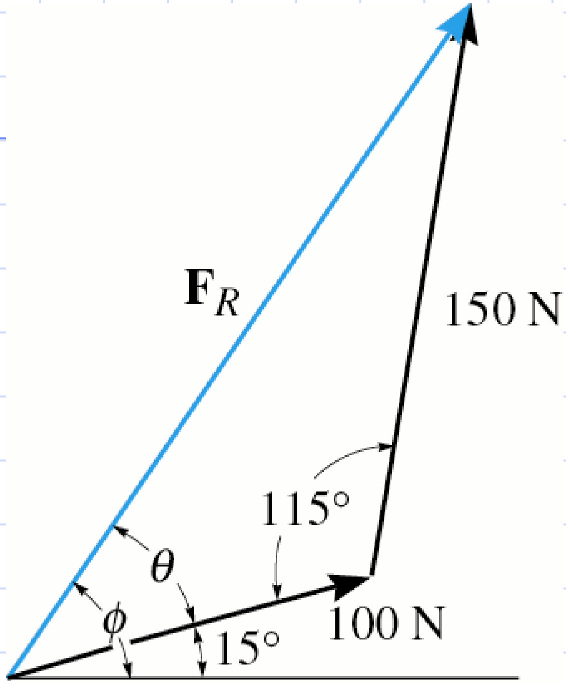
Find F_R from law of cosines.

Find θ from law of sines.

$$F_R = \sqrt{(100)^2 + (150)^2 - 2(100)(150)\cos 115^0}$$

$$F_R = \sqrt{10000 + 22500 - 30000(-0.4226)}$$

$$F_R = 212.6\,N = 213\,N$$



$$\frac{150}{\sin\theta} = \frac{212.6}{\sin 115^0}$$

$$\sin\theta = \frac{150}{212.6}(0.9063) = 0.6394$$

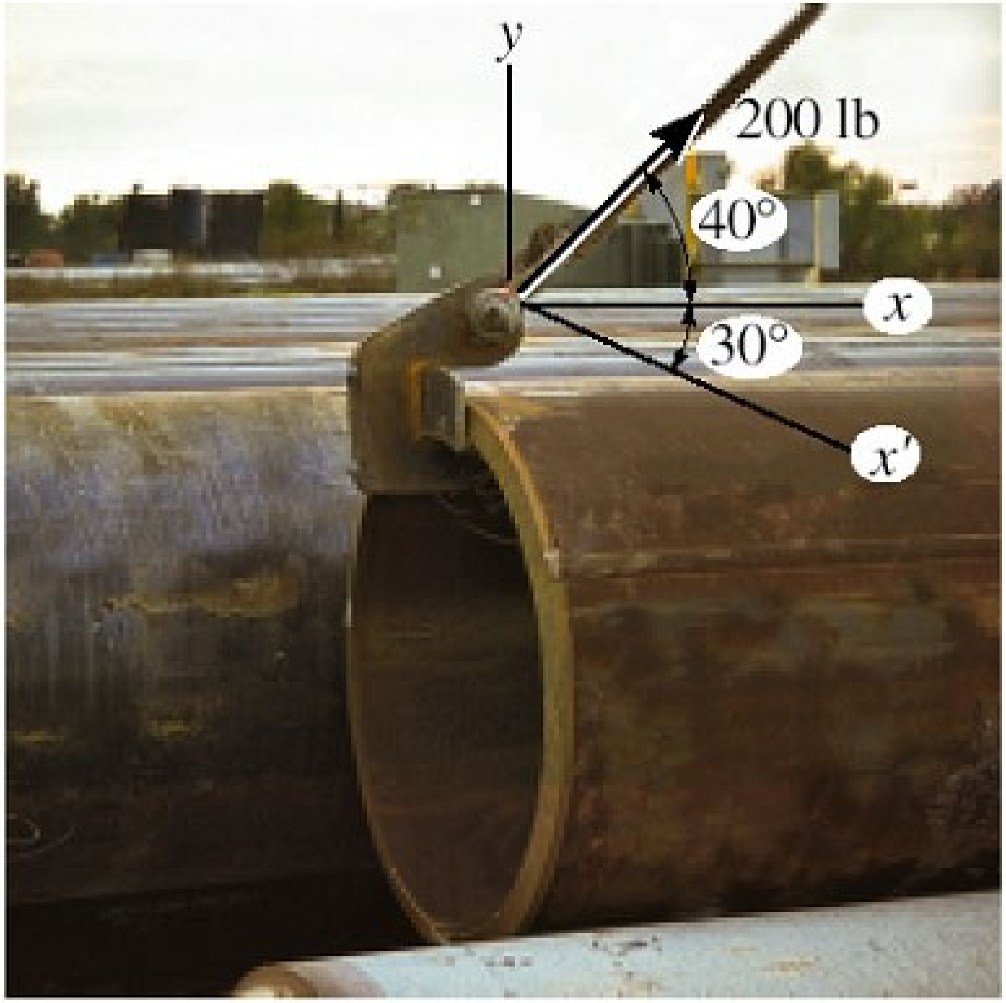
$$\theta = \sin^{-1}(0.6394) = 39.75^0 = 39.8^0$$

$$\phi = \theta + 15^0$$

Answer

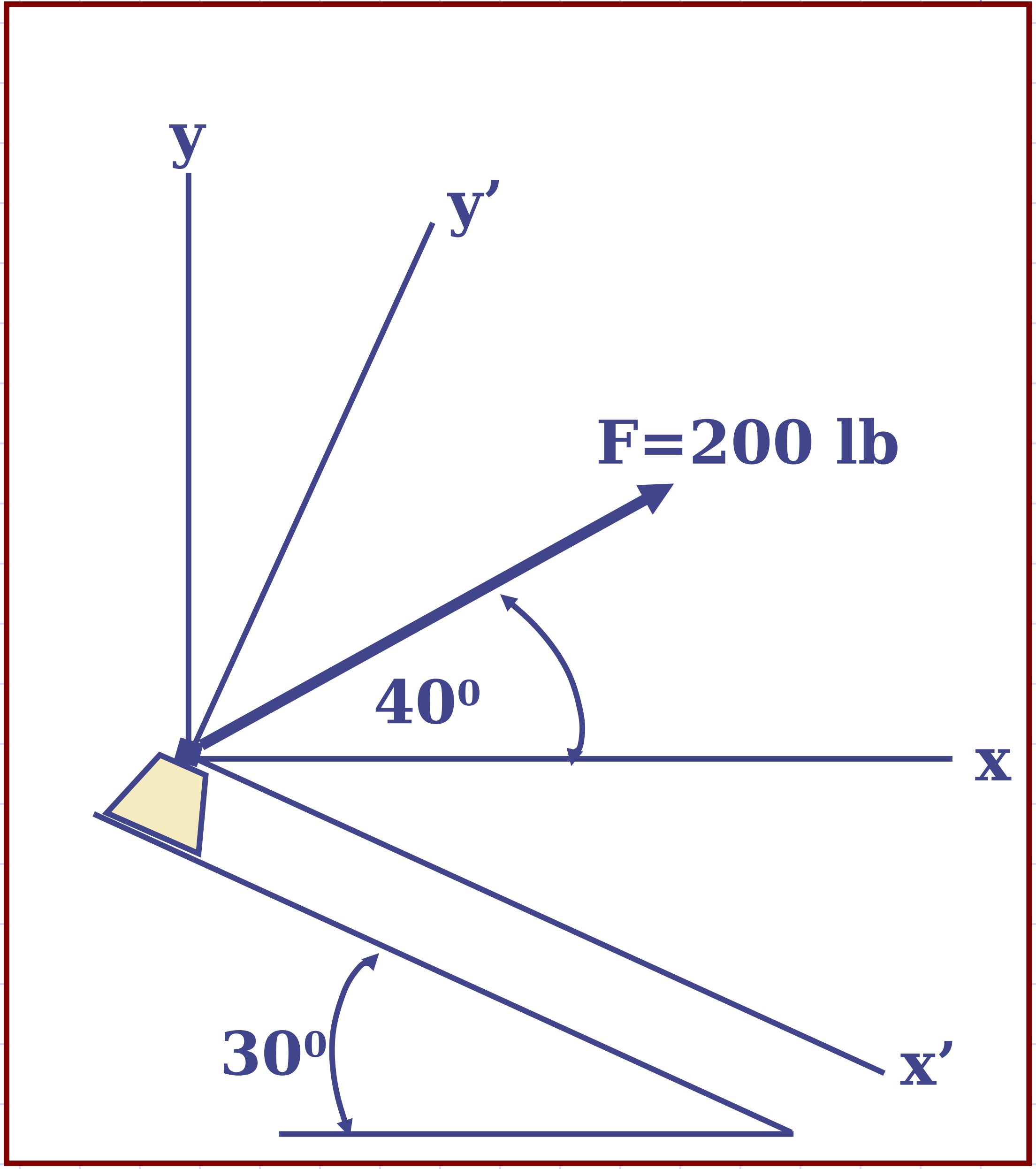
The resultant force has a magnitude of 213 N and is directed 54.8^{o} from the horizontal.

Example

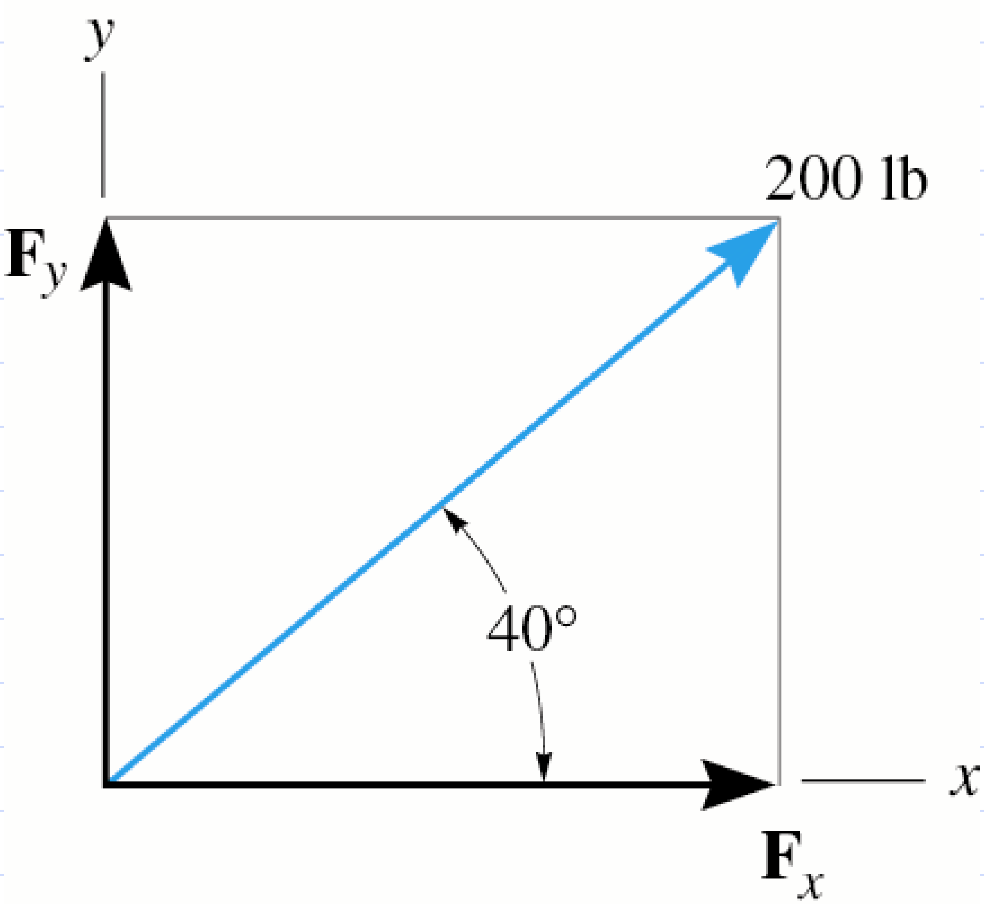


Resolve the 200 lb force into components in the x and y directions and in the x' and y directions

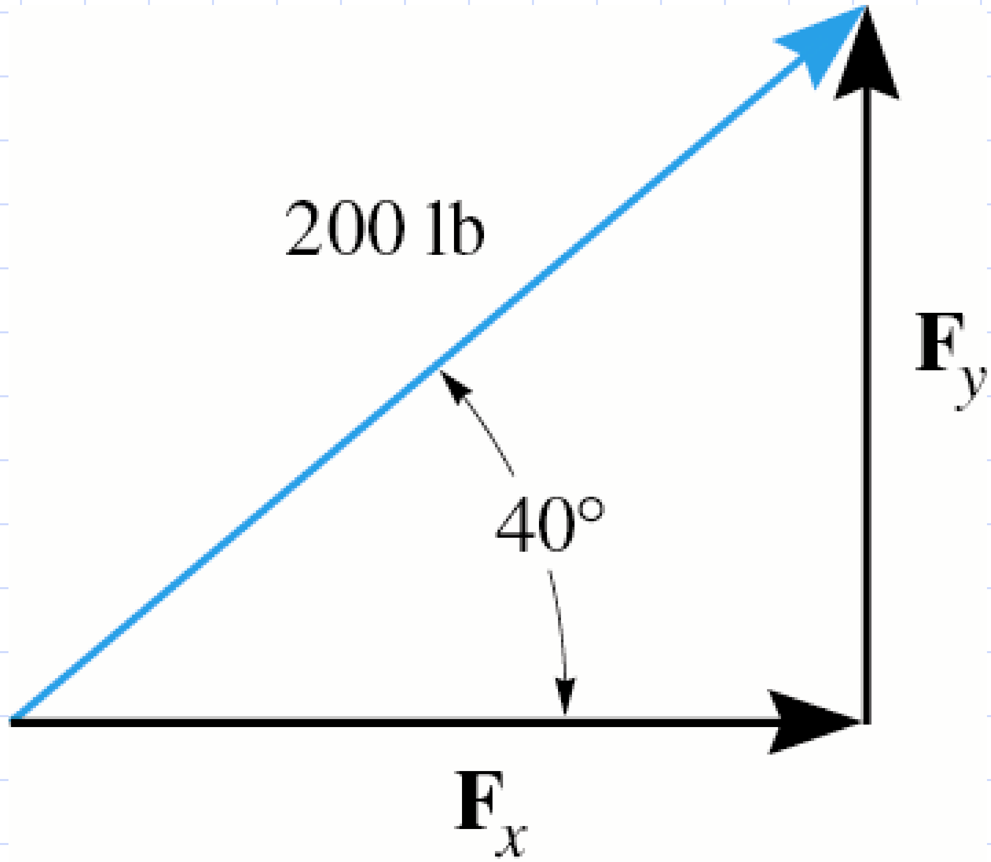
Resolve the 200 lb force into components in the x and y directions and in the x’ and y directions



Parallelogram Law



Triangular Construction



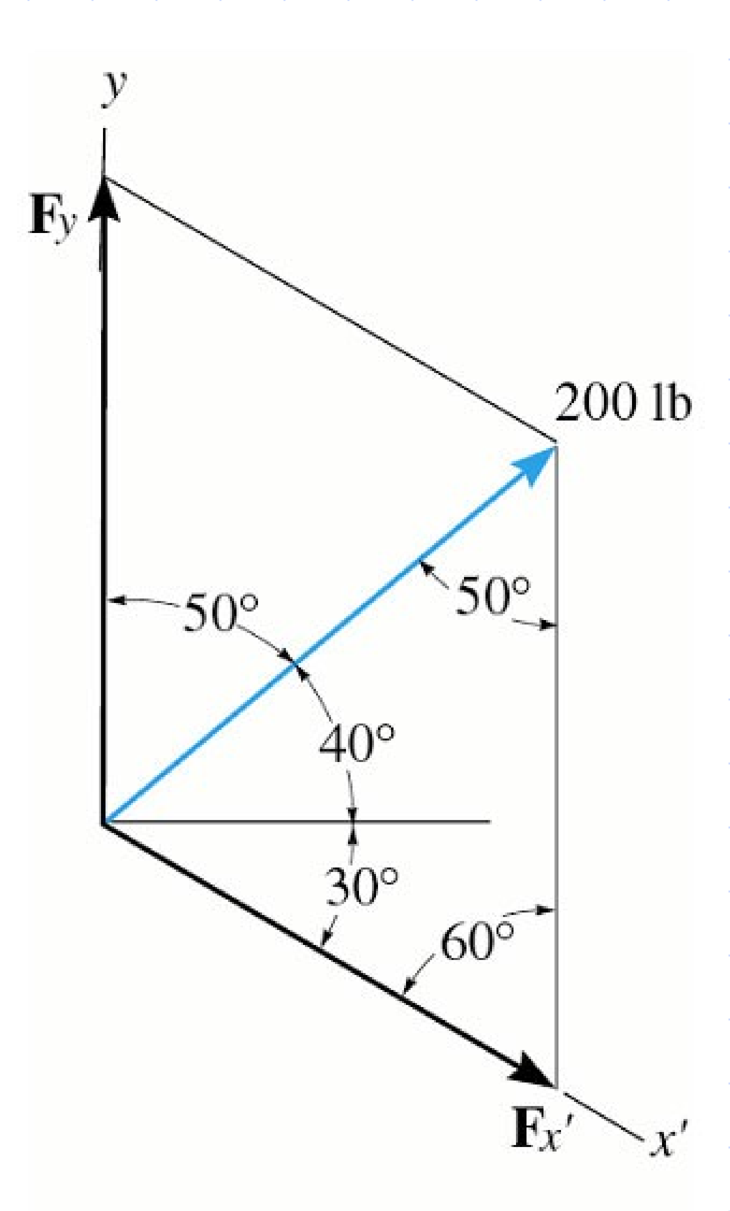
Solution – Part (a)

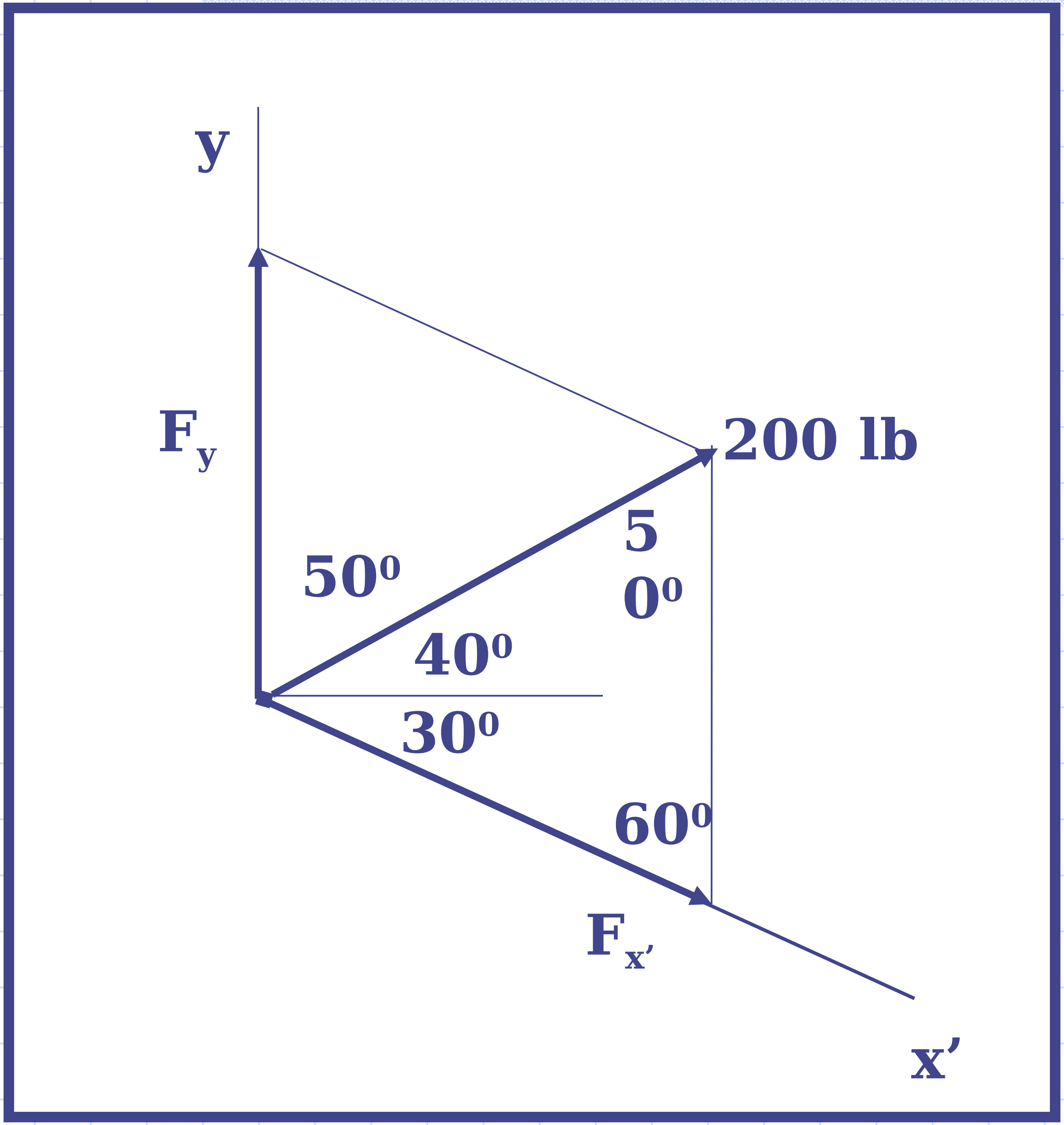
$$\vec{F} = \vec{F}_x + \vec{F}_y$$

$$F_x = 200\,\text{lb}\cos 40^o = 153\,\text{lb}$$

$$F_y = 200\,\text{lb}\sin 40^o = 129\,\text{lb}$$

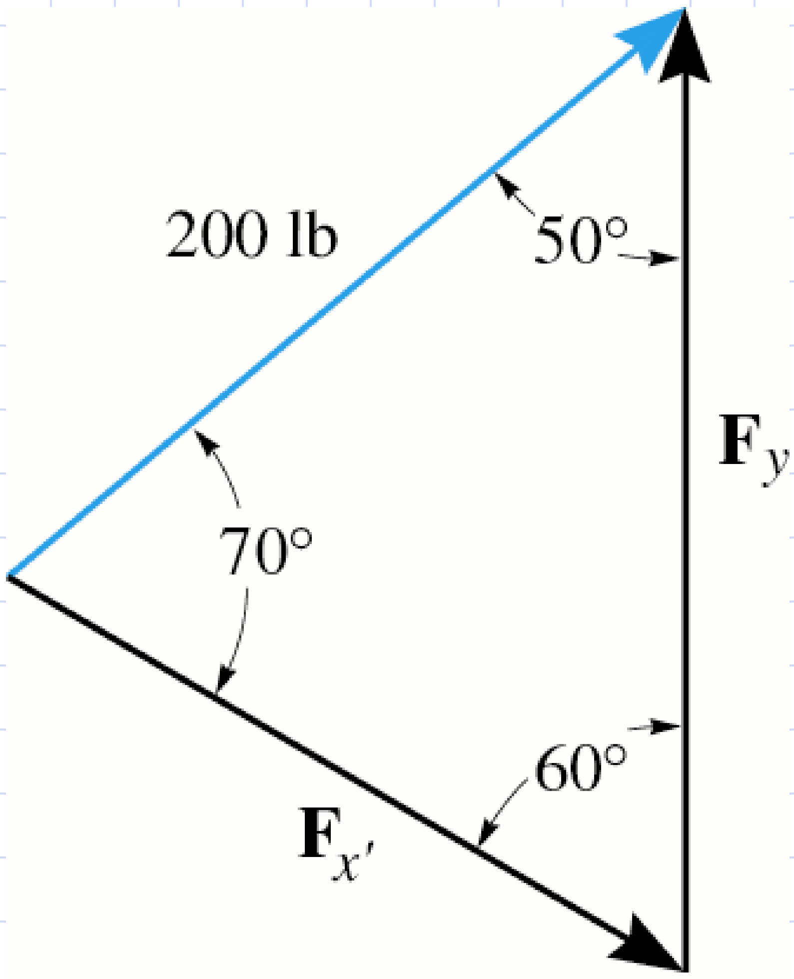
Parallelogram Law



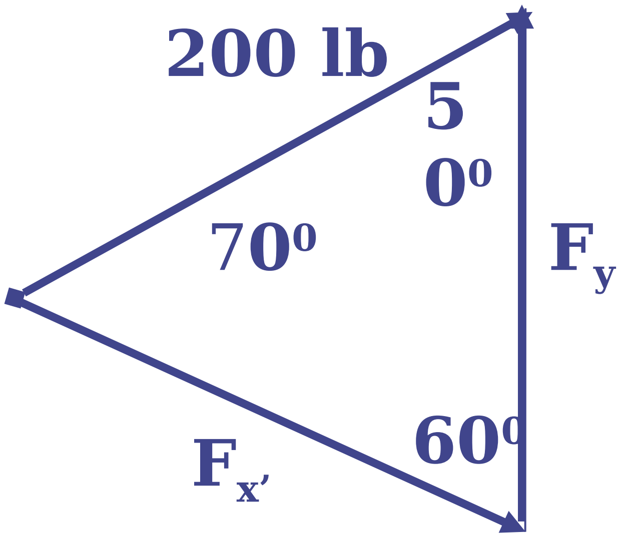


y
F
y
200 lb
5
0⁰
50⁰
40⁰
30⁰
60⁰
F
x'
x'

Triangular Construction



$$\boxed{\vec{F} = \vec{F}_{x'} + \vec{F}_y}$$



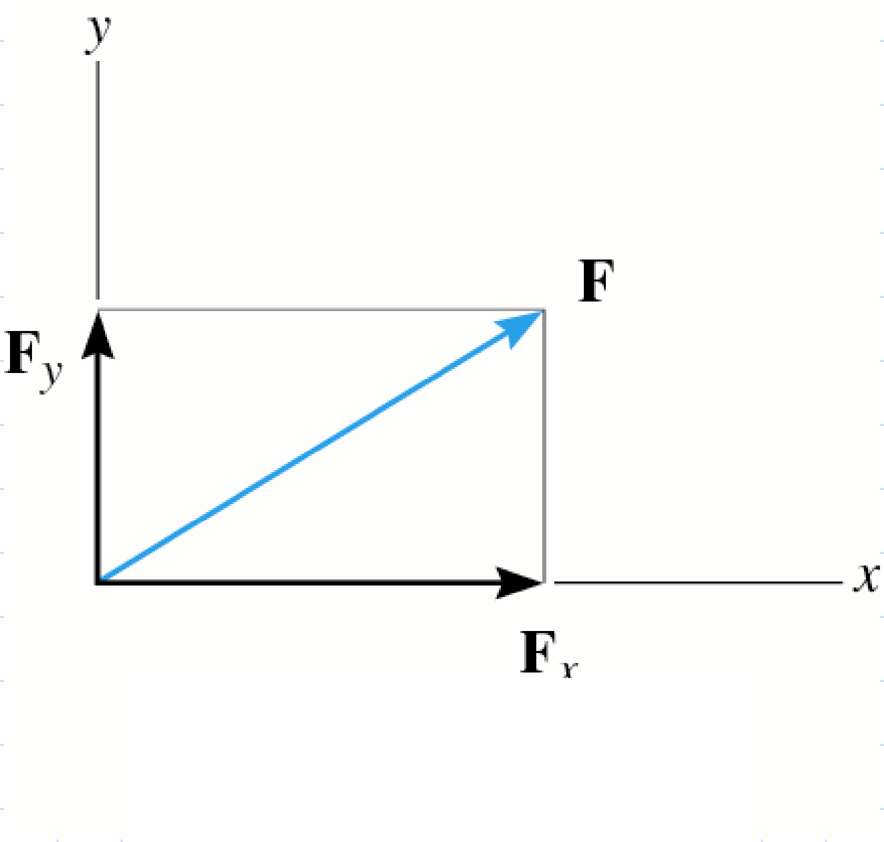
$$F_{x'} = 200\frac{\sin 50^0}{\sin 60^0} = 177\text{lb}$$

$$\frac{F_{x'}}{\sin 50^0} = \frac{200}{\sin 60^0}$$

$$F_y = 200\frac{\sin 70^0}{\sin 60^0} = 217\text{lb}$$

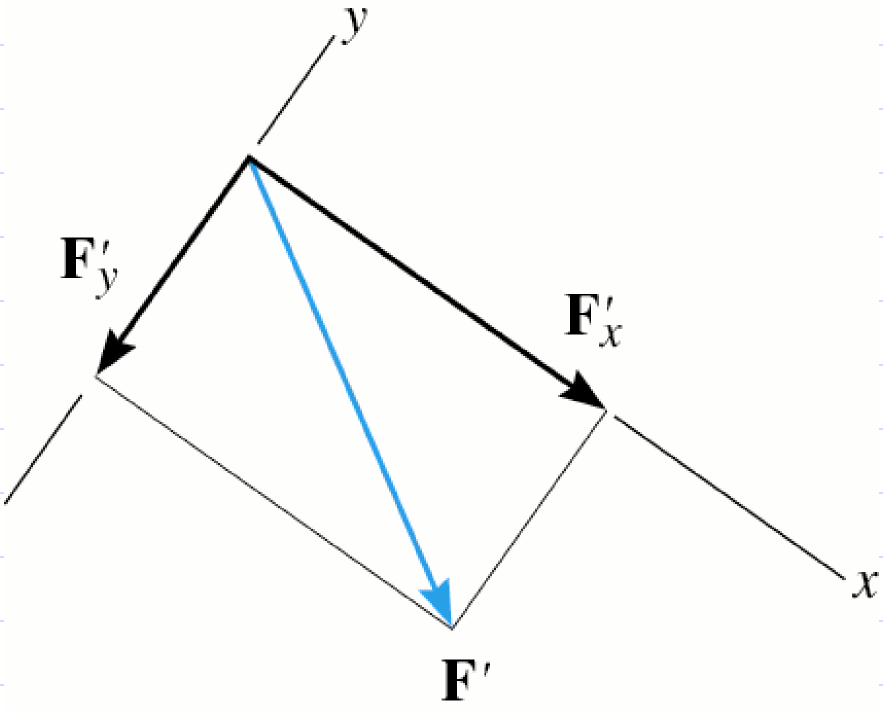
$$\frac{F_y}{\sin 70^0} = \frac{200}{\sin 60^0}$$

Addition of a System of Coplanar Forces



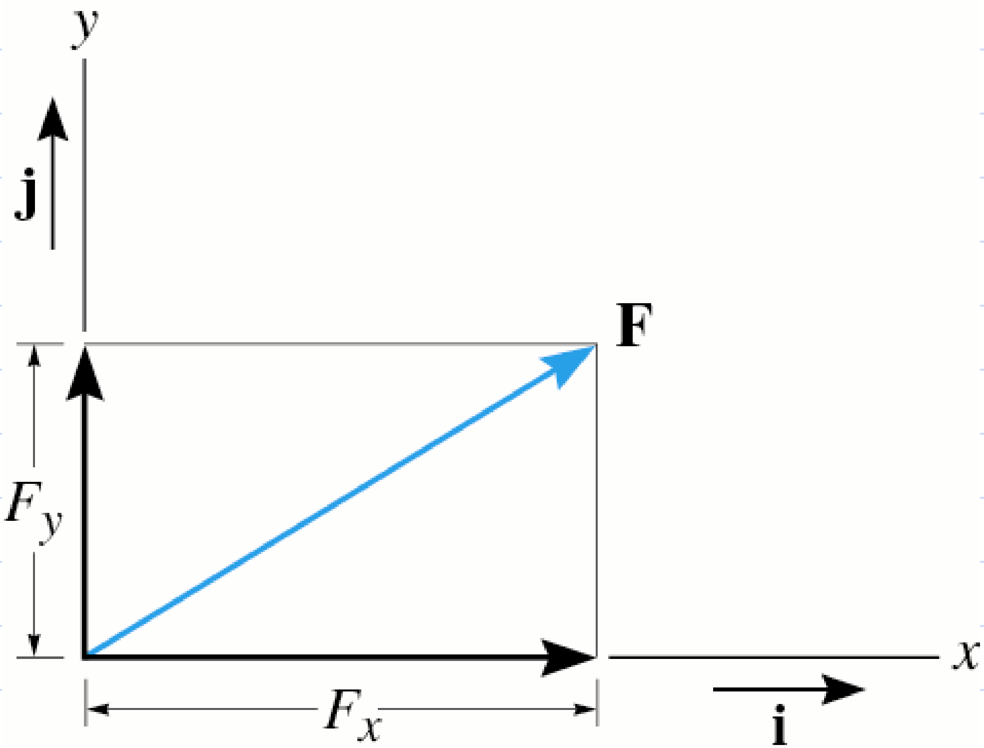
$$\vec{F} = \vec{F}_x + \vec{F}_y$$

Addition of a System of Coplanar Forces



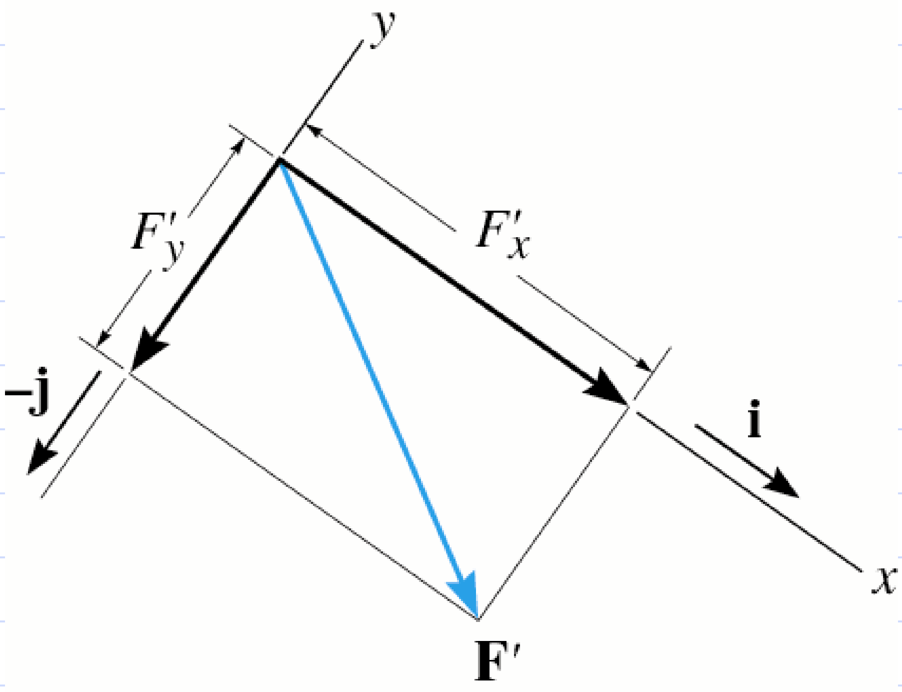
$$\vec{F}' = \vec{F}'_x + \vec{F}'_y$$

Cartesian Notation



$$\vec{F} = F_x\hat{i} + F_y\hat{j}$$

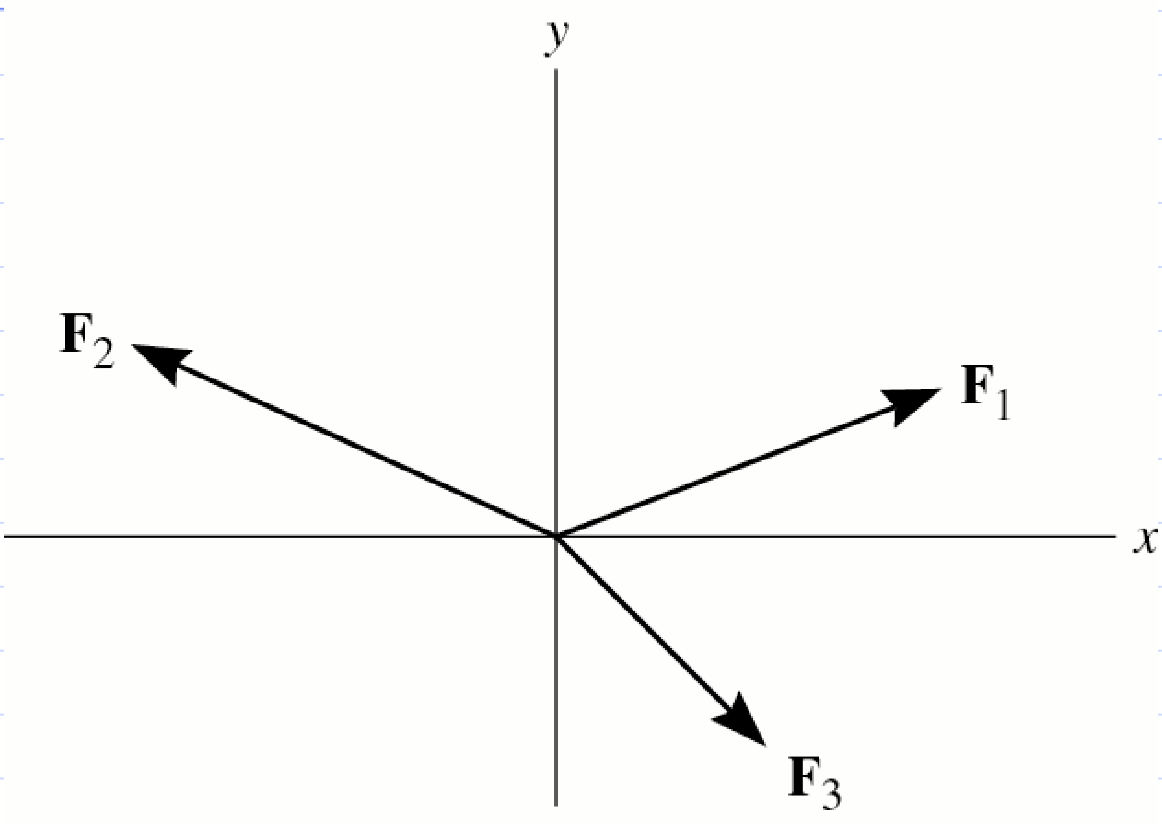
Cartesian Notation



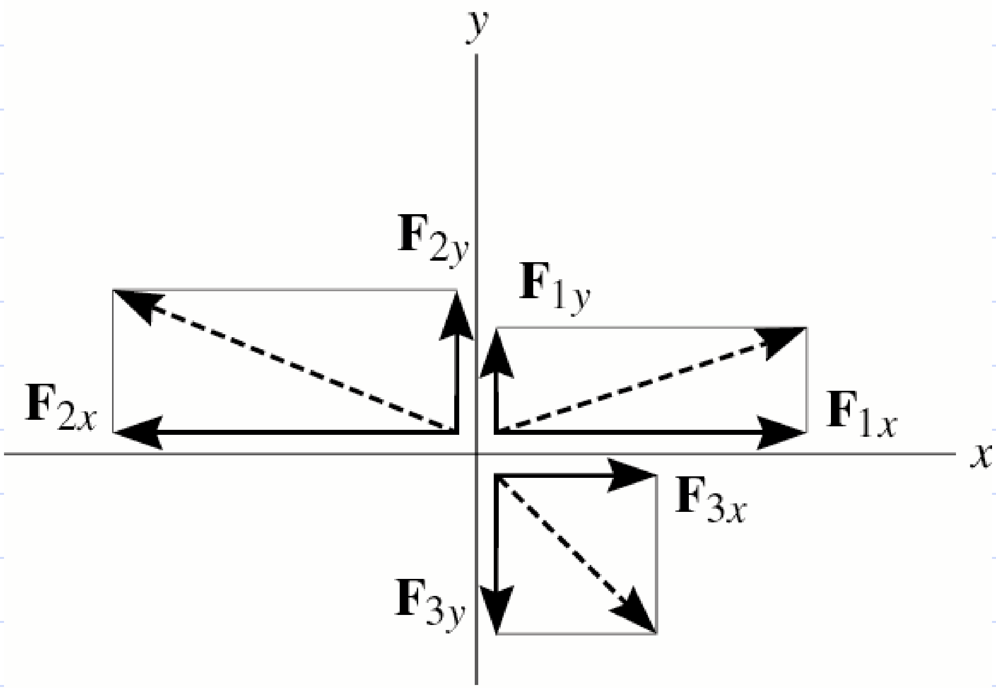
$$\vec{F}' = F'_x\hat{i} + F'_y\left(-\hat{j}\right)$$

$$\vec{F}' = F'_x\hat{i} - F'_y\hat{j}$$

Coplanar Force Resultants



Resolve into Cartesian Components



$$\vec{F}_1 = F_{1x}\hat{i} + F_{1y}\hat{j}$$

$$\vec{F}_2 = -F_{2x}\hat{i} + F_{2y}\hat{j}$$

$$\vec{F}_3 = F_{3x}\hat{i} - F_{3y}\hat{j}$$

Add Components

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\vec{F}_R = F_{1x}\hat{i} + F_{1y}\hat{j} - F_{2x}\hat{i} + F_{2y}\hat{j} + F_{3x}\hat{i} - F_{3y}\hat{j}$$

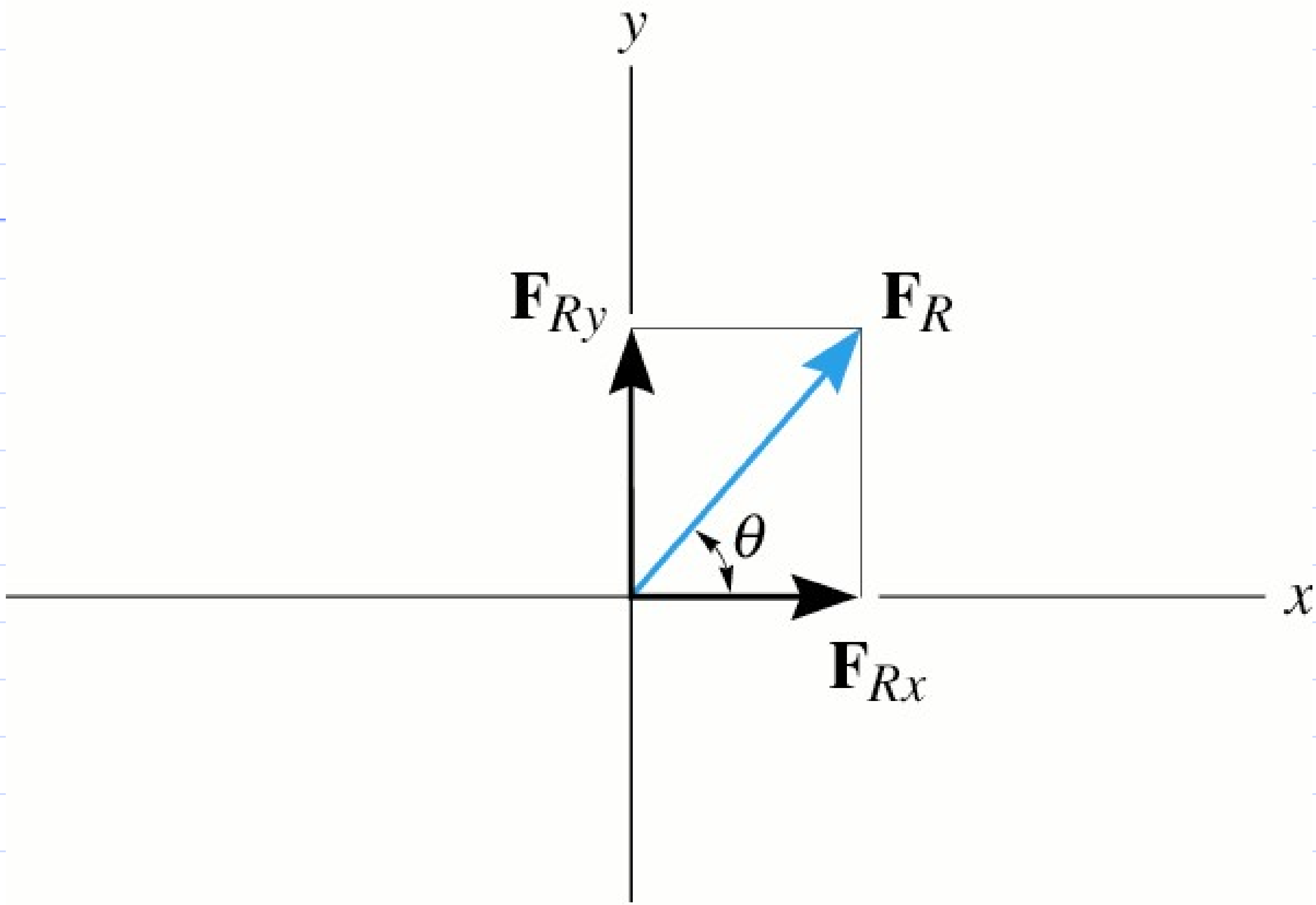
$$\vec{F}_R = F_{1x}\hat{i} - F_{2x}\hat{i} + F_{3x}\hat{i} + F_{1y}\hat{j} + F_{2y}\hat{j} - F_{3y}\hat{j}$$

$$\vec{F}_R = \left(F_{1x} - F_{2x} + F_{3x}\right)\hat{i} + \left(F_{1y} + F_{2y} - F_{3y}\right)\hat{j}$$

$$\vec{F}_R = F_{Rx}\hat{i} + F_{Ry}\hat{j}$$

$$F_{Rx} = \left(F_{1x} - F_{2x} + F_{3x}\right)$$

$$F_{Ry} = \left(F_{1y} + F_{2y} - F_{3y}\right)$$



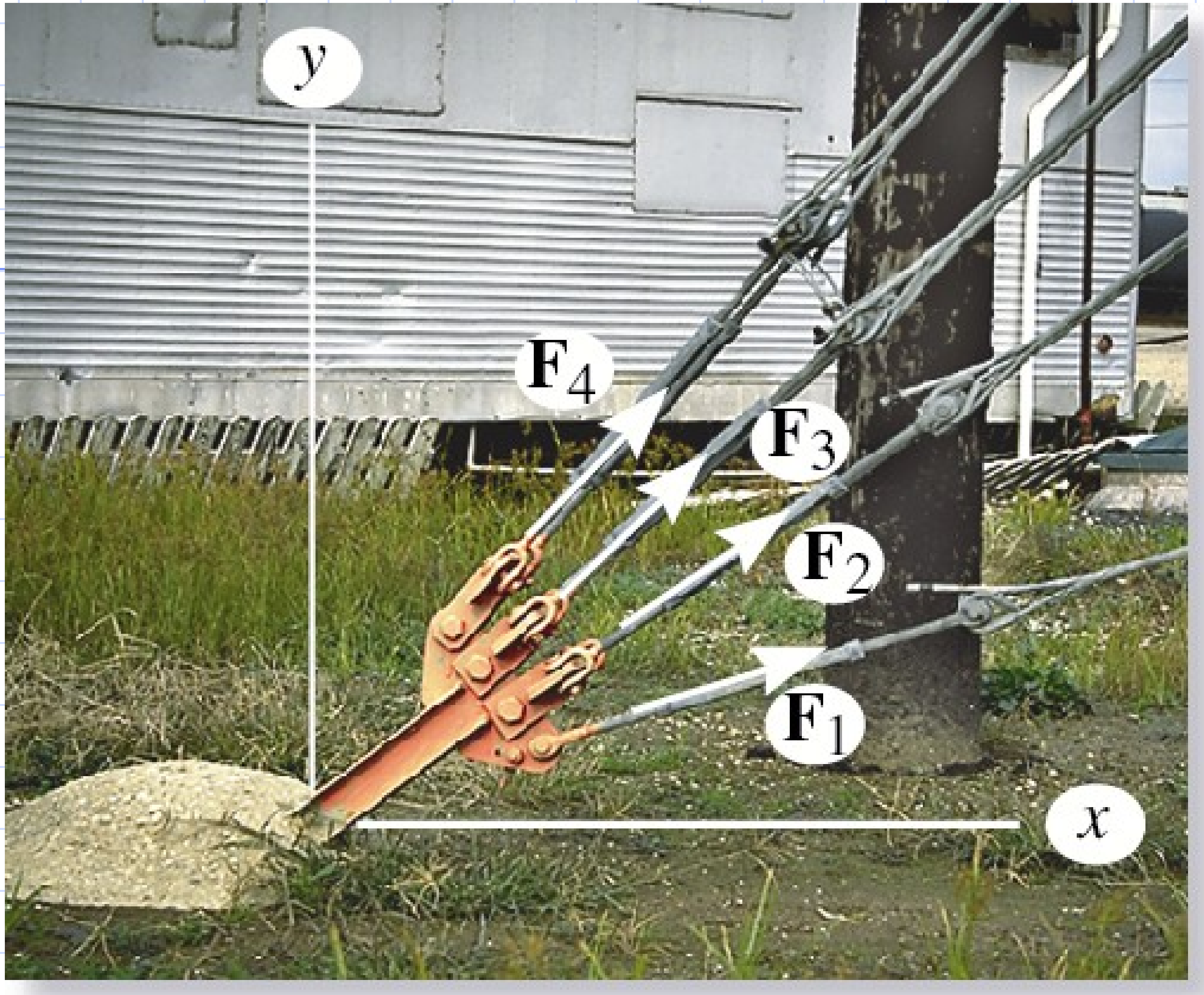
y
F_Ry
F_R
θ
x
F_Rx

$$F_{Rx} = \sum F_x$$

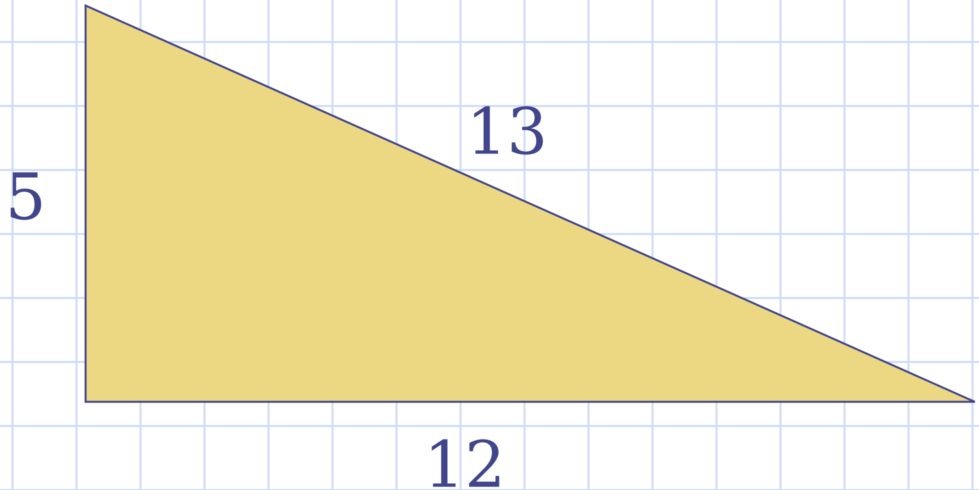
$$F_{Ry} = \sum F_y$$

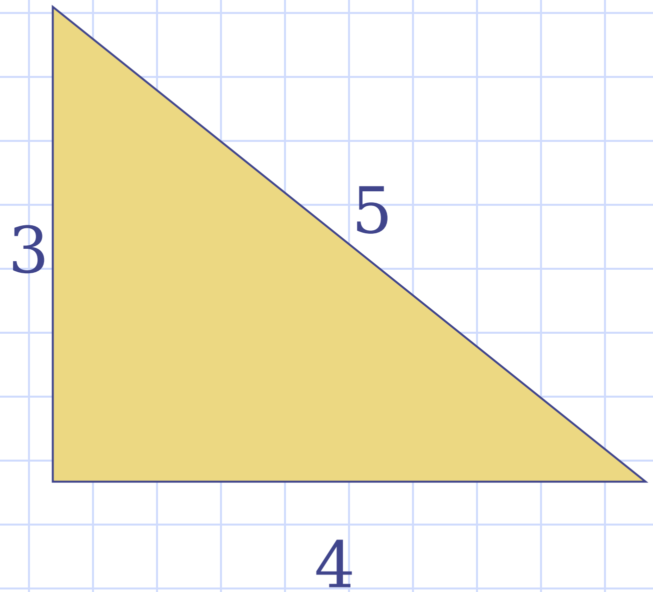
$$\left|\vec{F}_R\right| = F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

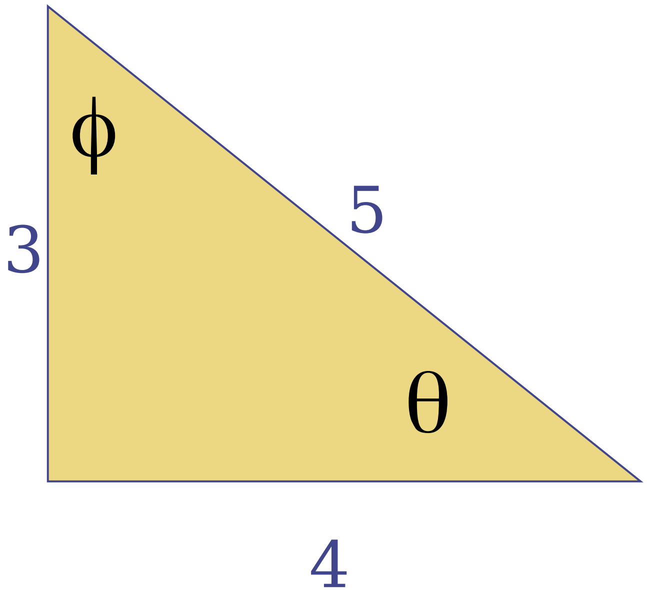
$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$


y
F4
F3
F2
F1
x

Special Triangles



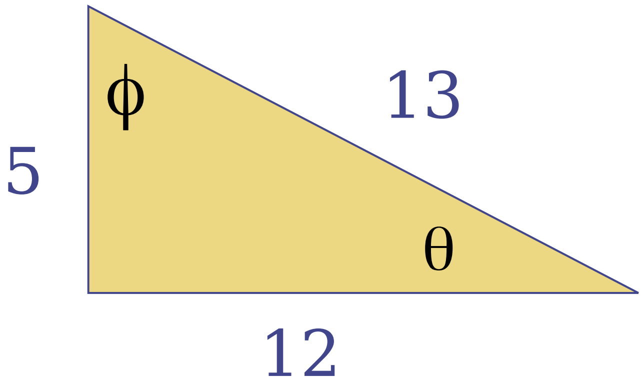




$$5 = \sqrt{3^2 + 4^2}$$

$$\cos\theta = \frac{4}{5} = 0.8 \qquad \sin\theta = \frac{3}{5} = 0.6$$

$$\cos\phi = \frac{3}{5} = 0.6 \qquad \cos\phi = \frac{4}{5} = 0.8$$

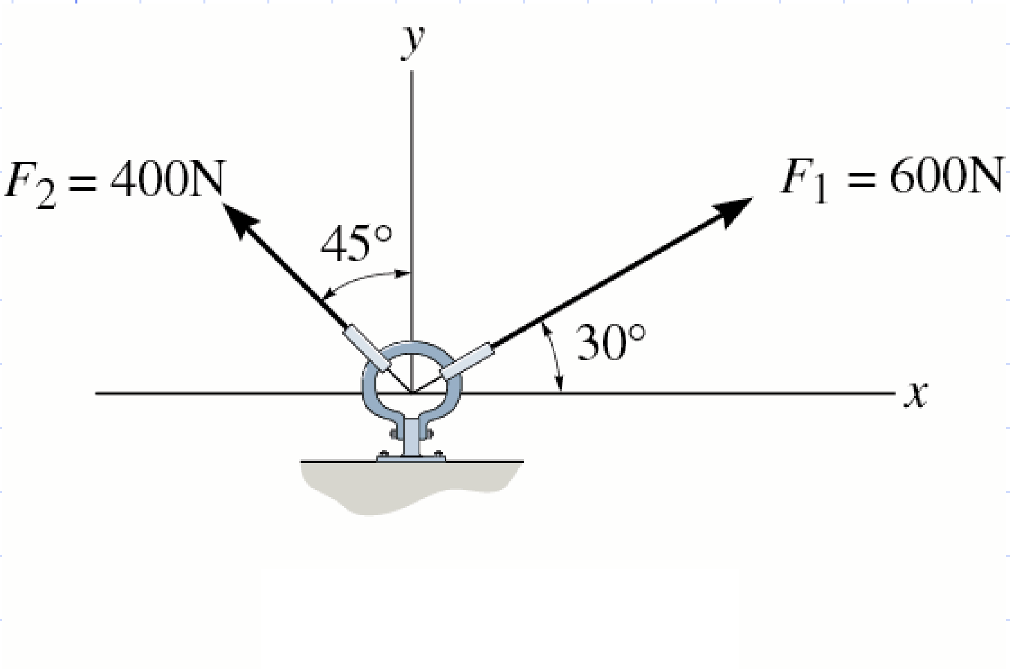


$$13 = \sqrt{5^2 + 12^2}$$

$$\cos\theta = \frac{12}{13} \qquad \sin\theta = \frac{5}{13}$$

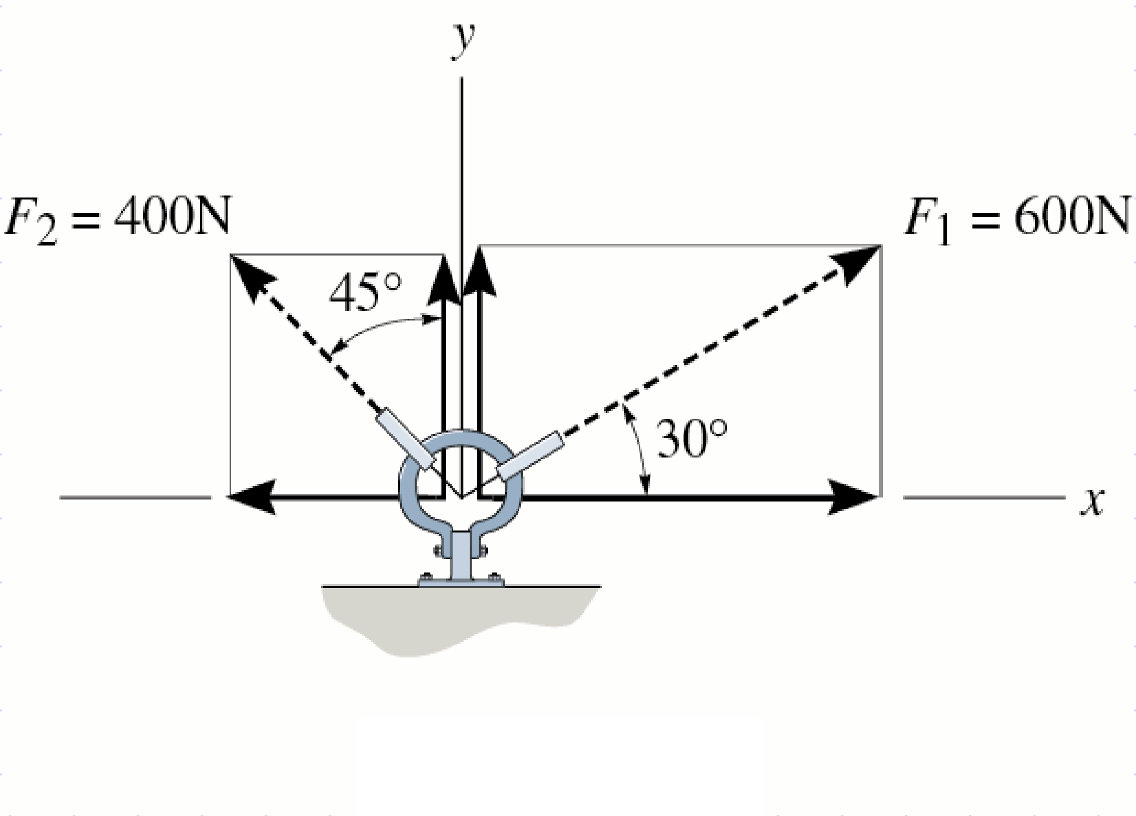
$$\cos\phi = \frac{5}{13} \qquad \cos\phi = \frac{12}{13}$$

Example



The link in the figure is subjected to two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the resultant magnitude and orientation of the resultant force.

Scalar Solution



Scalar Solution

$$\overset{+}{\rightarrow} \quad F_{R_x} = \sum F_x$$

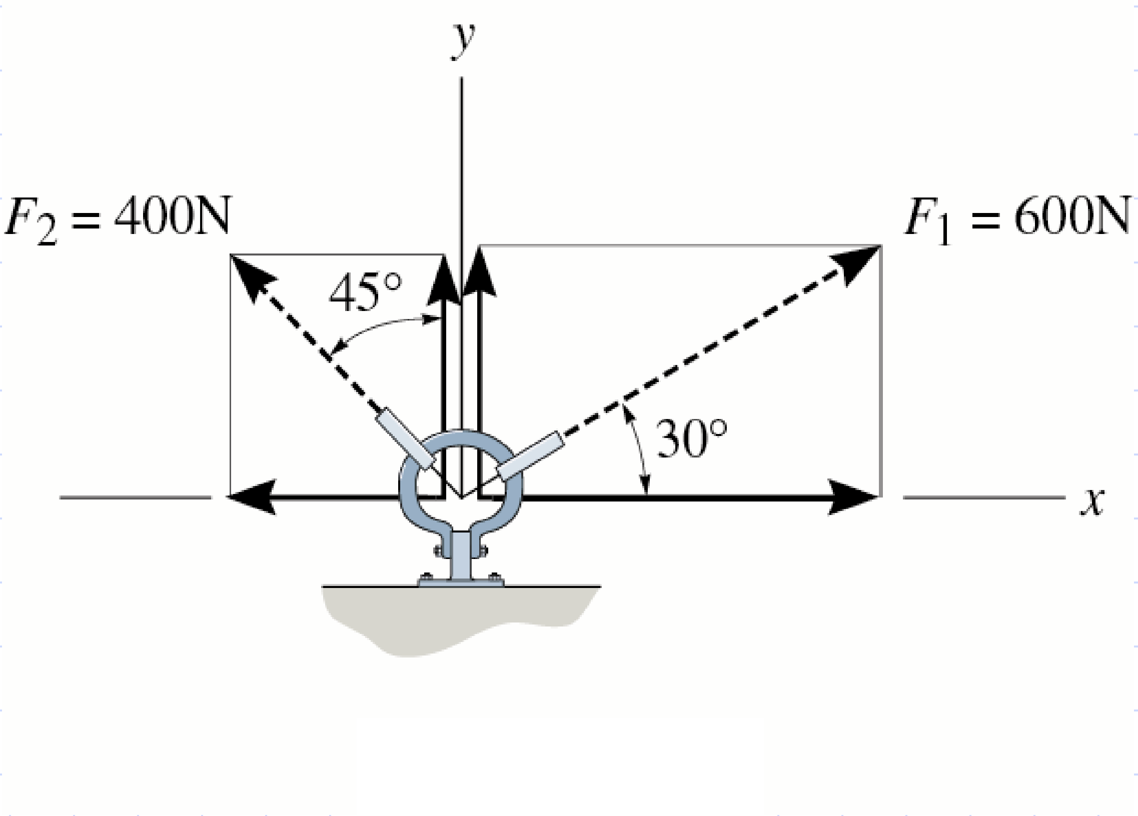
$$F_{R_x} = 600\cos 30^{o}\,N - 400\sin 45^{o}\,N = 236.8\,N \rightarrow$$

$$+\uparrow \quad F_{R_y} = \sum F_y$$

$$F_{R_y} = 600\sin 30^{o}\,N + 400\cos 45^{o}\,N = 582.8\,N \uparrow$$

$$\theta = \tan^{-1}\left(\frac{582.8\,N}{236.8\,N}\right) = 67.9^{o}$$

Cartesian Vector Solution



Cartesian Vector Solution

$$\vec{F}_1 = \left(600\cos 30^o\,\hat{i} + 600\sin 30^o\,\hat{j}\right)\text{N}$$

$$\vec{F}_2 = \left(600\cos 30^o\,\hat{i} + 600\sin 30^o\,\hat{j}\right)\text{N}$$

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$

$$= \left(600\cos 30^o\,\hat{i} + 600\sin 30^o\,\hat{j}\right)\text{N} +$$

$$\left(600\cos 30^o\,\hat{i} + 600\sin 30^o\,\hat{j}\right)\text{N}$$

$$\vec{F}_R = \left(236.8\,\hat{i} + 582.8\,\hat{j}\right)\text{N}$$